Application of Geometric Process for Generalized Exponential Distribution in Accelerated Life Testing with complete data

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Abstract: In this paper geometric process is used for the analysis of accelerated life testing under constant stress for the Generalized Exponential Distribution using complete data. By assuming that approach the lifetimes of units under increasing stress levels form a geometric process, the maximum likelihood estimation approached is used for the estimation of parameters. In order to get the asymptotic variance of the ML estimators, the Fisher information matrix is constructed. The asymptotic interval estimates of the parameters are then obtained by using this asymptotic variance. In the last, a simulation study is performed to illustrate the statistical properties of the parameters and the confidence intervals.

Keywords: Generalized Exponential Distribution, Geometric Process, Asymptotic Interval Estimate, Asymptotic Variance, Simulation Study.

1. Introduction

In modern era due to the development in design of products and manufacture in life cycles, the failures cannot be induced early at specified use conditions. Accelerated life testing is used to get early failures. It provides the details of testing items that is failure data (time) on the life distribution of materials or products. The test items are put under higher stress than normal usage condition. The model is fitted and extrapolated to estimate the life distribution under normal usage condition after getting the information by testing them at accelerated condition. This method is appropriate to test items of high reliability.

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The method is quicker and less costly than testing product at normal condition which is practically very difficult because of their long life.

Usually there are three types of stress loadings in accelerated life testing: constant stress, step stress and progressive stress (or linearly increasing stress). In constant stress test, each unit runs at prespecified constant stress levels which does not vary with time. This means that every unit is subjected to only one stress level untill the item fails or the test is stopped for any reason. Generally most products or items are assumed to operate at a constant stress when they are being used under normal conditions.

A lot of literature is available on constant stress accelerated life testing, for example, Ahmad et al. [1], Islam and Ahmad [2], Ahmad and Islam [4], Ahmad et al.[5] and Ahmad [6].Yang [7] proposed an optimal design of 4-level constant stress ALT plans considering different censoring times. Wilkins and Johns [8] considered constant stress accelerated life test based on Weibull distribution with constant shape and a log linear link between scale and the stress factor which is terminated by a Type-II censoring at one of the stress levels.

The geometric process concept was first introduced by Lam [9] in accelerated life testing in repair replacement problem. Lam [10] studied the geometric process model for a multistate system and concluded a replacement policy to minimize the long run average cost per unit time. After that a lot of work have been done and the available literature showing that a GP model is a good and simple model for analysis of data with a single trend or multiple trends, for example, Lam and Zhang[11], Lam[12] and Zhang[13].

Huang [14] did the analysis for exponential distribution with complete and censored data by using GP model. Zhou et al. [15] extended the GP model for the progressive type I hybrid censored Rayleigh failure data in ALT. Kamal et al. [16] analyzed constant stress accelerated life testing for Pareto distribution with complete samples by using geometric process model. Sadia Anwar et al.[17] used the mathematical model of accelerated life testing for Marshal-Olkin extended exponential distribution in geometric process, then extended her work in [18] for type I censored data.

The present study deals with the constant stress accelerated life testing for generalized Exponential distribution using geometric process with complete data. Estimates of parameters are obtained by maximum likelihood estimation technique and confidence intervals for parameters are obtained by using the asymptotic properties. Lastly, statistical properties of estimates and confidence intervals are examined through a simulation study.

2. The Model

2.1. The Geometric Process

A geometric process describes a stochastic process $\{X_n, n=1,2,...\}$, where there exists a real valued $\lambda > 0$ such that $\{\lambda^{n-1} X_n, n=1,2,...\}$ forms a renewal process. It can be shown that if $\{X_n, n=1,2,...\}$ is a GP and the probability density function of X_1 is $f(x)$ with mean μ and variance σ^2 then the probability density function of X_n will be $\lambda^{n-1} f(\lambda^{n-1}x)$ with $E(X_n)=\frac{\mu}{\lambda^{n-1}}$ and $\mathrm{var}(X_n)=\frac{\sigma^2}{\lambda^{2(n-1)}}$. Thus λ, μ and σ^2 are three important parameters of GP.

2.2. The Generalized Exponential Distribution

The probability density function (pdf) of a generalized exponential distribution is given by

$$f(x,\alpha,\beta)=\frac{\alpha}{\beta}e^{-\frac{x}{\beta}}\left(1-e^{-\frac{x}{\beta}}\right)^{\alpha-1}, \qquad x>0$$

Where $\alpha > 0$, is the shape parameter and $\beta > 0$, is the scale parameter of the distribution. GE distribution with the shape parameter α and the scale parameter β will be denoted by $GE(\alpha,\beta)$. $GE(1,\beta)$ represents the exponential distribution with the scale parameter β.

It is observed that the two-parameter generalized exponential distribution can be used quite electively in analyzing many lifetime data, particularly in place of two-parameter gamma and two-parameter Weibull distributions [3]. Depending on the shape parameter, it can have an increasing and decreasing failure rates. The cumulative distribution function (cdf) of generalized exponential distribution is

$$F(x,\alpha,\lambda)=\left(1-e^{-\frac{x}{\beta}}\right)^{\alpha}, \quad \alpha,\lambda,x>0$$

The survival function of the generalized exponential distribution takes the following form

$$S(x,\alpha,\lambda)=1-\left(1-e^{-\frac{x}{\beta}}\right)^{\alpha}$$

The failure rate or hazard rate is given by

$$h(x,\alpha,\lambda)=\frac{\frac{\alpha}{\beta}e^{-\frac{x}{\beta}}\left(1-e^{-\frac{x}{\beta}}\right)^{\alpha-1}}{1-\left(1-e^{-\frac{x}{\beta}}\right)^{\alpha}}$$

The shape of the hazard function depends only on shape parameter α. It can be observed that the generalized exponential distribution has a log-concave density for α > 1 and it is log-convex for α ≤ 1. Therefore for the fixed value of scale parameter β, the generalized exponential distribution has an increasing hazard function for α > 1 and it has a decreasing hazard function for α < 1. For α = 1, it has constant hazard function. The hazard function of the generalized exponential distribution behaves exactly the same way as the hazard functions of the gamma distribution, which is quite different from the hazard function of the Weibull distribution [4].

2.3. Assumptions and test procedure

1. Suppose that we conduct an accelerated life test with *s* increasing stress levels. Under each stress level, a random sample of *n* identical items is placed and start to operate at the same time. Let x_{ki}, $i = 1,2,...,n \quad k = 1,2,...,s$ be the observed failure time of *ith* test item under *kth* stress level. The failed item will be removed from the test and test would be continued till all the test items get failed (complete data).
2. The product life follows generalized exponential distribution denoted by $GE(\alpha,\beta)$ at any stress.
3. The scale parameter is a log-linear function of stress. That is, $\log(\beta_k) = a + bS_k$, where *a* and *b* are unknown parameters depending on the nature of the product and the test method.
4. Let random variables $X_0, X_1, X_2,..., X_S$, denote the lifetimes under each stress level, where X_0 denotes item's lifetime under the design stress at which items will operate ordinarily and sequence $\{X_k, k = 1,2,...s\}$ forms a geometric process with ratio $\lambda > 0$.

The assumptions above discussed are very common in ALT literature except the last one, i.e. assumption 4. It is the assumption of geometric process which is better than the usual one without increasing the complexity in calculations. The assumption 4 can be shown by the following theorem assuming that there is a log linear relationship between a life and stress (assumption 3).

Theorem: If the stress level in an ALT is increasing with a constant difference then under each stress level the life times of items forms a GP. That is, If $S_{k+1} - S_k$ is constant for $k = 1,2,..., s-1$, then $\{X_k, k = 1,2,..., s\}$ forms a GP.

Proof: From assumption (3), we get

$$\log\left(\frac{\beta_{k+1}}{\beta_k}\right) = b\left(S_{k+1} - S_k\right) = b\Delta S$$

This shows that the increased stress levels form an arithmetic sequence with a constant difference ΔS . Now the above equation can be written as

$$\frac{\beta_{k+1}}{\beta_k} = e^{b\Delta S} = \frac{1}{\lambda}(say) \qquad (1)$$

It is clear from (1) that

$$\beta_k = \frac{1}{\lambda}\beta_{k-1} = \frac{1}{\lambda^2}\beta_{k-2} = ... = \frac{1}{\lambda^k}\beta$$

The lifetime pdf of an item at the kth stress level is

$$f_{X_k}(x) = \frac{\alpha}{\beta_k} e^{\frac{-x}{\beta_k}}\left(1 - e^{\frac{-x}{\beta_k}}\right)^{\alpha-1}$$

$$= \frac{\alpha\lambda^k}{\beta} e^{\frac{-\lambda^k x}{\beta}}\left(1 - e^{\frac{-\lambda^k x}{\beta}}\right)^{\alpha-1} \qquad (2)$$

And the cdf is

$$F_{X_k}(x) = (1 - e^{\frac{-x\lambda^k}{\beta}})^{\alpha} \qquad (3)$$

Eq.(2) implies that

$$f_{X_k}(x) = \lambda^k f_{X_0}\left(\lambda^k x\right) \qquad (4)$$

Now, from the definition of GP and from expression (4) it is clear that, if density functions of X_0 is $f_{X_0}(x)$, then the pdf of X_k will be given by $\lambda^k f_{X_0}\left(\lambda^k x\right)$, $k = 1,2,..., s$. Therefore, it is clear that lifetimes under a sequence of arithmetically increasing stress levels form a GP with ratio λ .

The expression (2) shows that if lifetimes of items under a sequence of increasing stress level form a geometric process with ratio λ and if the life distribution at design stress level is generalized exponential with characteristic β, then the life distribution at k^{th} stress level will also be generalized exponential with characteristic life $\frac{\lambda^k}{\beta}$.

3. Maximum Likelihood Estimation

Maximum likelihood estimation (MLE) is one of the most widely used methods among all estimation methods. It can be applied to any probability distribution while other methods are somewhat restricted. MLE implementation in ALT is mathematically more complex and, generally, closed form estimates of parameters do not exist. Therefore, numerical techniques such as Newton Raphson method is used to compute them. The likelihood function for constant stress ALT for complete case generalized exponential failure data using GP for s stress levels is given by:

$$L = \prod_{k=1}^{s}\prod_{i=1}^{n} \frac{\alpha\lambda^k}{\beta} e^{\frac{-\lambda^k x}{\beta}} \left(1 - e^{\frac{-\lambda^k x}{\beta}}\right)^{\alpha-1} \quad (5)$$

The log-likelihood function corresponding to above can be rewritten as;

$$l = \sum_{k=1}^{s}\sum_{i=1}^{n} \begin{bmatrix} log\alpha + klog\lambda - log\beta \\ -\frac{\lambda^k x}{\beta} + \\ (\alpha-1)\log\left(1 - e^{\frac{-\lambda^k x}{\beta}}\right) \end{bmatrix} \quad (6)$$

The MLEs of α, β and λ are obtained by solving the normal equations $\frac{\partial l}{\partial \alpha} = 0, \frac{\partial l}{\partial \beta} = 0$ and $\frac{\partial l}{\partial \lambda} = 0$ as follows;

$$\frac{\partial l}{\partial \alpha} = \frac{ns}{\alpha} + \sum_{k=1}^{s}\sum_{i=1}^{n} \log(1-z) = 0 \quad (7)$$

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$$\frac{\partial l}{\partial \beta} = \frac{-ns}{\beta} + \sum_{k=1}^{s}\sum_{i=1}^{n} \begin{bmatrix} \frac{\lambda^k x}{\beta^2} - \frac{\lambda^k x}{\beta^2} \times \\ (\alpha-1)\frac{Z}{(1-Z)} = 0 \end{bmatrix} = 0 \quad (8)$$

$$\frac{\partial l}{\partial \lambda} = \frac{kns}{\lambda} - \sum_{k=1}^{s}\sum_{i=1}^{n} \begin{bmatrix} \frac{k\lambda^{(k-1)} x}{\beta} \\ -k\lambda^{k-1} x \frac{(\alpha-1)}{\beta}\frac{Z}{1-Z} \end{bmatrix} = 0 \quad (9)$$

Where,

$$Z = e^{-\frac{\lambda^k x}{\beta}}$$

Equations (7), (8) and (9) are used to find the estimate of α, β and λ

4. Fisher Information Matrix & Asymptotic Confidence Interval

The asymptotic Fisher Information matrix is given by:

$$\begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \beta} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 l}{\partial \beta \partial \alpha} & -\frac{\partial^2 l}{\partial \beta^2} & -\frac{\partial^2 l}{\partial \beta \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda \partial \beta} & -\frac{\partial^{2l}}{\partial \lambda^2} \end{bmatrix}$$

The elements of Fisher Information matrix can be obtained by putting a minus sign before double and partial derivatives of the parameters, which are given as follows;

$$\frac{\partial^2 l}{\partial \alpha^2} = -\frac{ns}{\alpha^2}$$

$$\frac{\partial^2 l}{\partial \beta^2} = \frac{ns}{\beta^2} - \sum_{k=1}^{s}\sum_{i=1}^{n} \begin{bmatrix} \frac{2x\lambda^2}{\beta^3} + (\alpha-1)x\lambda^2 \times \\ \left\{\frac{Zx\lambda^k - 2\beta Z(1-Z)}{\beta^4(1-Z)}\right\} \end{bmatrix}$$

$$\frac{\partial^2 l}{\partial \lambda^2} = -\frac{kns}{\lambda^2} - \sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{k(k-1)x\lambda^{k-2}}{\beta} - \frac{kxZ(\alpha-1)}{\beta} \times \left\{\frac{(k-1)\lambda^{k-2}}{(1-Z)} - \frac{xk\lambda^{2(k-1)}}{\beta(1-Z^2)}\right\}\right]$$

$$\frac{\partial^2 l}{\partial \alpha \partial \beta} = \frac{\partial^2 l}{\partial \beta \partial \alpha} = -\sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{\lambda^k xZ}{\beta^2(1-Z)}\right]$$

$$\frac{\partial^2 l}{\partial \alpha \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \alpha} = \sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{kx\lambda^{k-1}Z}{\beta(1-Z)}\right]$$

$$\frac{\partial^2 l}{\partial \beta \partial \lambda} = \frac{\partial^2 l}{\partial \lambda \partial \beta} = \sum_{k=1}^{s}\sum_{i=1}^{n}\left[\frac{kx\lambda^{k-i}}{\beta^2} - \frac{(\alpha-1)Zkx}{\beta^2} \times \left\{\frac{\lambda^{k-1}}{(1-Z)} - \frac{x\lambda^{2k-1}}{\beta(1-Z)^2}\right\}\right]$$

Now the variance-covariance matrix can be written as

$$\Sigma = \begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \beta} & -\frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 l}{\partial \beta \partial \alpha} & -\frac{\partial^2 l}{\partial \beta^2} & -\frac{\partial^2 l}{\partial \beta \partial \lambda} \\ -\frac{\partial^2 l}{\partial \lambda \partial \alpha} & -\frac{\partial^2 l}{\partial \lambda \partial \beta} & -\frac{\partial^2 l}{\partial \lambda^2} \end{bmatrix}^{-1}$$

The asymptotic confidence interval for α, β and λ are given by following expressions:

$\hat{\alpha} + Z_{1-\frac{\phi}{2}}(SE(\hat{\alpha})), \hat{\beta} + Z_{1-\frac{\phi}{2}}(SE(\hat{\beta}))$, and $\hat{\lambda} + Z_{1-\frac{\phi}{2}}(SE(\hat{\lambda}))$ respectively.

5. Simulation Study:

Simulation of data is the initial task for studying different properties of parameters. It is an attempt to model an assumed condition to study the behaviour of function.

To perform the simulation study, first a random sample is generated from Uniform distribution by using R software.

Now we use inverse cdf method to transform eq(3) in terms of u and get the expression of x_{ki}, k=1,2,...,s and i=1,2,...,n.

$$x_{ki} = -\beta.\frac{\log\left(1-u^{\frac{1}{\alpha}}\right)}{\lambda^k} \quad , k = 1,2,\ldots,s \;\; i = 1,2,\ldots,n$$

- 1000 random samples of size 20,40,60,80 and 100 have been obtained from the generalized exponential distribution.
- The values of parameters and numbers of the stress levels are chosen to be α=1.2, β=2.8, λ=1.1 and s=4 or 6.
- By using optim() function, we obtain ML estimates, the mean squared error(MSE), relative absolute bias(RAB), relative error(RE) and lower and upper bound of 95% and 99% confidence intervals for different sample sizes.

The results obtained in the above simulation study are summarized in Table1 & 2.

6. Conclusion:

In this study, geometric process is introduced for the analysis of accelerated life testing under constant stress when the life data are from a Generalized exponential model. It is better choice for life testing because of its simplicity in nature. The Mean, SE, MSE, RAB and RE of the parameters are obtained . Based on the asymptotic normality, the 95% and 99% confidence intervals of the parameters are also obtained.

The results in Table 1 and Table 2 show that the estimated values of α, β and λ are very close to true (or initial) values with very small SE and MSE. As sample size increases, the value of SE and MSE decreases and the confidence interval become narrower.

For the Table 2, the maximum likelihood estimators have good statistical properties than the Table 1 for all sample sizes.

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Table 1: Simulation results of Generalised exp using GP at α=1.2, β=2.8, λ=1.1 and s=4

Sample	Estimates	Mean	SE	√MSE	RABias	RE	Lower BOund	Upper Bound
20	α	1.0772	0.1990	0.1923	0.1022	0.1602	0.6870 0.5636	1.4674 1.5908
	β	3.0781	0.3191	0.0950	0.0993	0.0339	2.4526 2.2547	3.7036 3.9015
	λ	1.0500	0.1034	0.0999	0.0908	0.0864	0.7971 0.7329	1.2028 1.2670
40	α	1.1675	0.1200	0.1159	0.02707	0.0966	0.9322 0.8578	1.4027 1.4771
	β	3.0397	0.2565	0.0614	0.0856	0.0219	2.5369 2.3779	3.5424 3.7015
	λ	0.9952	0.1035	0.1000	0.0909	0.0558	0.7970 0.7329	1.2028 1.2670
60	α	1.2113	0.1201	0.1160	0.0094	0.0967	0.9758 0.9014	1.4467 1.5212
	β	3.0038	0.2417	0.0545	0.0728	0.0194	2.5299 2.3800	3.4777 3.6276
	λ	1.0000	0.1034	0.0999	0.0908	0.0495	0.7971 0.7330	1.2028 1.2670
80	α	1.1826	0.0805	0.0777	0.0144	0.0648	1.0248 0.9749	1.3404 1.3904
	β	3.0491	0.2645	0.0653	0.0889	0.0233	2.5306 2.3666	3.5676 3.7316
	λ	0.9924	0.1035	0.1000	0.0909	0.0593	0.7970 0.7328	1.2029 1.2671
100	α	1.1579	0.0867	0.0838	0.0350	0.0698	0.9878 0.9340	1.3280 1.3818
	β	3.0515	0.2661	0.0661	0.0898	0.0236	2.5299 2.3649	3.5731 3.7381
	λ	1.0041	0.1035	0.0999	0.0909	0.0600	0.7971 0.7329	1.2028 1.2670

Table 2: Simulation results of Generalised exp using GP at α=1.2, β=2.8, λ=1.1 and s=6

Sample	Estimates	Mean	SE	√MSE	RABias	RE	Lower BOund	Upper Bound
20	α	1.2838	0.1409	0.1361	0.0698	0.1134	1.0076 0.9202	1.5601 1.6474
	β	2.9798	0.2183	0.0444	0.0642	0.0158	2.5518 2.4165	3.4077 3.5431
	λ	0.9950	0.1035	0.1000	0.0909	0.0404	0.7970 0.7328	1.2029 1.2671
40	α	1.1598	0.1451	0.1402	0.0334	0.1168	0.8753 0.7853	1.4442 1.5342
	β	3.0598	0.2899	0.0784	0.0928	0.0280	2.4916 2.3118	3.6281 3.8079
	λ	1.0489	0.1035	0.1000	0.0909	0.0713	0.7971 0.7329	1.2028 1.2670
60	α	1.2019	0.0968	0.0935	0.0015	0.0779	1.0120 0.9519	1.3917 1.4517
	β	3.0650	0.2777	0.0719	0.0946	0.0257	2.5207 2.3485	3.6092 3.7814
	λ	0.9923	0.1035	0.1000	0.0909	0.0654	0.7970	1.2028

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							0.7329	1.2670
80	α	1.1868	0.0892	0.0861	0.0110	0.0718	1.0119 0.9566	1.3616 1.4169
	β	3.0600	0.2708	0.0684	0.0928	0.0244	2.5291 2.3612	3.5909 3.7588
	λ	0.9973	0.1035	0.1000	0.0909	0.0622	0.7970 0.7328	1.2029 1.2671
100	α	1.1977	0.0812	0.0785	0.0019	0.0654	1.0384 0.9880	1.3570 1.4073
	β	3.0587	0.2718	0.0689	0.0923	0.0246	2.5260 2.3574	3.5914 3.7600
	λ	1.0500	0.1035	0.1	0.0909	0.0626	0.7971 0.7329	1.2028 1.2670

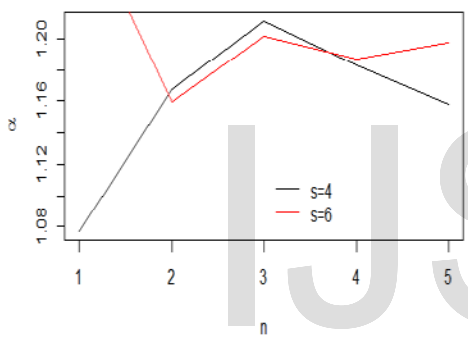

Fig.1 Estimate of alpha of the GE model at different stresses

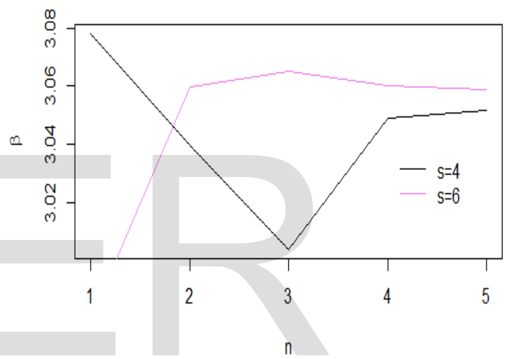

Fig.2 Estimate of the beta for GE model at different stresses

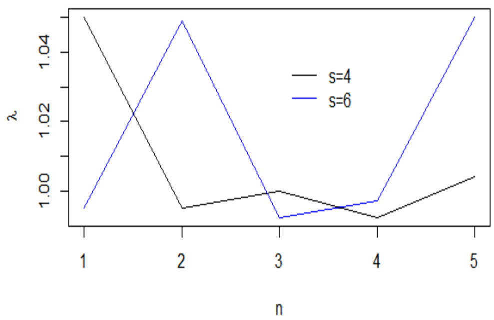

Fig.3 Estimate of the lambda for GE model at different stresses

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